Section 5.1 Anti-derivatives / Integration (Minimum Homework: 1 – 31 odds)

Integration (finding the antiderivative) is the process of finding a function, given its derivative.

> Each problem in this section will give us the derivative of a function.
>
> We will need to find the function whose derivative gives the function presented in the problem.

These are at least three different ways to ask the question; What function has the derivative of a given function:

The three statements below all ask the same question using different words or symbols.

- Find the function whose derivative is $f'(x)$.
- Given $f'(x), find\ f(x)$
- Find $\int(f(x))dx$

The third example uses a new symbol that we call ab integral sign.

> Notation in the third example:
>
> $\int f(x)dx$ = asks us to find the function whose derivative is $f(x)$
>
> - The **indefinite integral** sign ∫ represents **integration**.
> - The **symbol** $dx\ is$ called the differential of the variable x, indicates that the variable of **integration** is x.
> - The function $f(x)$ to be **integrated** is called the integrand.

Example: Let us try to use the indefinite integration symbol to solve a simple integration problem.

$\int(2x)dx$

This is asking me to find a function $f(x)$ that has a derivative of $2x$

This is sneaky.

$f(x) = x^2$ is certainly a function that has the property that $f'(x) = 2x$.

Unfortunately, it is not the only function that has the property that $f'(x) = 2x$

Each of these functions also has the property that:

$f'(x) = 2x\ (as\ the\ constant\ has\ a\ derivative\ of\ 0)$

$f(x) = x^2 + 3$
$f(x) = x^2 - 5$
$f(x) = x^2 + 12$
$f(x) = x^2 - 1$

There are infinitely many functions that have a derivative of $2x$.

This is the way we will write the answer to take into consideration that there are infinitely many solutions:

Answer: $\int(2x)dx = x^2 + C\ (where\ C\ is\ any\ real\ number\ (constant))$

This allows infinitely many answers of the same form.

Basic Integration Rules

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int a dx = ax + C \quad (a\ is\ any\ real\ number)$

"ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

"e" Rule $\int e^x dx = e^x + C$

$C\ represents\ any\ real\ number$

Properties of integrals

$\int \big(f(x) + g(x)\big)dx = \int f(x)dx + \int g(x)dx$

$\int \big(f(x) - g(x)\big)dx = \int f(x)dx - \int g(x)dx$

$\int af(x)dx = a\int f(x)dx$

Let us practice these rules by finding the following integrals.

$\int 12x^3dx$ $\int 9dx$ $\int(8x^3 - 9x^2 + 2)dx$

$\int 8x(x^3 + 3x - 4)dx$ $\int \frac{7x^2-5x}{2x}dx$

$\int -5e^x dx$ $\int \frac{12}{x}dx$ $\int \frac{2}{3}x^{-1}dx$

Problem: $\int 12x^3dx$

Rewrite: Use this rule to rewrite $\int af(x)dx = a\int f(x)dx$

$12\int x^3dx$

Integration: Use the power rule $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

$= 12 * \frac{1}{4}x^{3+1} + C$
$= 3x^4 + C$

Answer: $3x^4 + C$

Problem: $\int 9dx$

Rewrite: None needed

Integration: Integral of a constant Rule: $\int adx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

$\int 9dx$ = $9x + C$

Answer: $9x + C$

Problem: $\int (8x^3 - 9x^2 + 2)dx$

Rewrite: First use these properties of integrals:

$\int \big(f(x) + g(x)\big)dx = \int f(x)dx \quad + \int g(x)dx$

$\int \big(f(x) - g(x)\big)dx = \int f(x)dx \quad - \int g(x)dx$

$= \int 8x^3 dx \;\; - \int 9x^2 dx \; + \;\; \int 2dx$

Next apply $\int af(x)dx = a\int f(x)dx$

$= 8\int x^3 dx - 9\int x^2 dx + \int 2dx$

Integration:
Rules needed

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int adx \;=\; ax + C \quad (a\ is\ any\ real\ number)$

$= 8 * \frac{1}{4}x^4 - 9 * \frac{1}{3}x^3 + 2x + C$

$= 2x^4 - 3x^3 + 2x + C$

Answer: $2x^4 - 3x^3 + 2x + C$

Problem: $\int 8x(x^3 + 3x - 4)dx$

$Rewrite$

First use basic Algebra to rewrite.

$= \int (8x^4 + 24x^2 - 32x)dx$

Then apply:

$\int \big(f(x) + g(x)\big)dx = \int f(x)dx \quad + \int g(x)dx$

$\int \big(f(x) - g(x)\big)dx = \int f(x)dx \quad - \int g(x)dx$

$= \int 8x^4dx + \int 24x^2dx \ - \int 32xdx$

Then apply:

Next apply $\int af(x)dx = a\int f(x)dx$

$= 8\int x^4dx + 24\int x^2dx \ - 32\int xdx$

Integration:

Only need the Power rule
Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

$= 8 * \frac{1}{5}x^5 + 24 * \frac{1}{3}x^3 - 32 * \frac{1}{2}x^2 + C$

= $\frac{8}{5}x^5 + 8x^3 - 16x^2 + C$

Answer: = $\frac{8}{5}x^5 + 8x^3 - 16x^2 + C$

Problem: $\int \frac{7x^2-5x}{2x}dx$

Rewrite:

First use basic Algebra

$$\int \frac{7x^2-5x}{2x}dx = \int \left(\frac{7x^2}{2x} - \frac{5x}{2x}\right)dx = \int \left(\frac{7}{2}x - \frac{5}{2}\right)dx$$

Now apply: $\int (f(x) - g(x))dx = \int f(x)dx \quad - \int g(x)dx$

$$= \int \frac{7}{2}xdx \quad - \int \frac{5}{2}dx$$

Next apply $\int af(x)dx = a\int f(x)dx$

$$= \frac{7}{2}\int xdx - \int \frac{5}{2}\ dx$$

Integrate:

Rules needed:

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int adx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

$$= \frac{7}{2} * \frac{1}{2}x^2 - \frac{5}{2}x + C$$

$$= \frac{7}{4}x^2 - \frac{5}{2}x + C$$

Answer: $= \frac{7}{4}x^2 - \frac{5}{2}x + C$

Problem: $\int -5e^x dx$

Rewrite

Next apply $\int af(x)dx = a\int f(x)dx$

$= -5\int e^x dx$

Integrate:

Rule needed:

"e" Rule $\int e^x dx \ = e^x + C$

$= -5e^x + C$

Answer: $-5e^x + C$

Problem: $\int \frac{12}{x} dx$

Rewrite:

Basic Algebra

$= \int \left(12 * \frac{1}{x}\right) dx$

Next apply $\int af(x)dx = a \int f(x)dx$

$= 12 \int \frac{1}{x} dx$

Integrate:

Use: "ln" Rule: $\left\{ \int \frac{1}{x} dx = \ln|x| + C \right.$

$= 12ln|x| + C$

Answer: $= 12ln|x| + C$

Problem: $\int \frac{2}{3} x^{-1} dx$

Rewrite:

Apply: $\int af(x)dx = a \int f(x)dx$

$= \frac{2}{3} \int x^{-1} dx$

Integrate:

Apply

"ln" Rule: $\left\{ \int x^{-1} dx = \ln|x| + C \right.$

$= \frac{2}{3} ln|x| + C$

Answer: $= \frac{2}{3} ln|x| + C$

#1-32: Find the following antiderivatives, be sure to include the plus "C" in your answer. (minimum homework – all odds)

1) $\int 3x^2 dx$

2) $\int 2x^3 dx$

1st: $\int af(x)dx = a\int f(x)dx$

2nd: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

$Answer \quad \int 2x^3 dx = \frac{1}{2}x^4 + C$

3) $\int \frac{1}{3} x dx$

4) $\int \frac{1}{2} x dx$

1st: $\int af(x)dx = a \int f(x)dx$

2nd: Power Rule: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ n \neq -1$

Answer: $\int \frac{1}{2} x dx = \frac{1}{4} x^2 + C$

5) $\int 2dx$

6) $\int 7dx$

1st: Integral of a constant Rule: $\int adx \;=\; ax + C \quad (a\ is\ any\ real\ number)$

Answer: $\int 7dx = 7x + C$

7) $\int (6x + 5)dx$

8) $\int (4x^3 - 3x^2 + 5)dx$

1st:

$\int (f(x) + g(x))dx = \int f(x)dx \quad + \int g(x)dx$

$\int (f(x) - g(x))dx = \int f(x)dx \quad - \int g(x)dx$

2nd: $\int af(x)dx = a \int f(x)dx$

3rd:

Power Rule: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int adx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

Answer: $\int (4x^3 - 3x^2 + 5)dx = x^4 - x^3 + 5x + C$

9) $\int \frac{5}{x^2} dx$

10) $\int \frac{3}{x^4} dx$

1st: Rewrite with negative exponent

2nd: $\int af(x)dx = a\int f(x)dx$

3rd: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

4th: rewrite with positive exponent

Answer: $\int \frac{3}{x^4} dx = -\frac{1}{x^3} + C$

11) $\int \frac{3}{x^4} dx$

12) $\int \frac{8}{x^3} dx$

1st: Rewrite with negative exponent

2nd: $\int af(x)dx = a\int f(x)dx$

3rd: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

4th: rewrite with positive exponent

Answer: $\int \frac{8}{x^3} dx = -\frac{4}{x^2} + C$

13) $\int 2x(x^2+3)dx$

14) $\int 4x(x^3+3x-4)dx$

1st: Rewrite by clearing parenthesis

2nd:

$\int \big(f(x)+g(x)\big)dx = \int f(x)dx \quad + \int g(x)dx$

$\int \big(f(x)-g(x)\big)dx = \int f(x)dx \quad - \int g(x)dx$

3rd: $\int af(x)dx = a\int f(x)dx$

4th :

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Answer: $\int 4x(x^3+3x-4)dx = \frac{4}{5}x^5 + 4x^3 - 8x^2 + C$

15) $\int (3x+4)^2 dx$

16) $\int (2x+5)^2 dx$

1st: Rewrite as a FOIL problem and simplify

2nd:

$\int \big(f(x)+g(x)\big)dx = \int f(x)dx \quad + \int g(x)dx$

3rd: $\int af(x)dx = a\int f(x)dx$

4th :

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int adx \; = \; ax + C \quad (a\ is\ any\ real\ number)$

$Answer$: $\int (2x+5)^2 dx = \frac{4}{3}x^3 + 10x^2 + 25x + C$

17) $\int (x+1)\,(x-4)dx$

18) $\int (x-4)\,(x+7)dx$

1st: Rewrite as a FOIL problem and simplify

2nd:

$\int \big(f(x)+g(x)\big)dx = \int f(x)dx \quad + \int g(x)dx$

$\int \big(f(x)-g(x)\big)dx = \int f(x)dx \quad - \int g(x)dx$

3rd: $\int af(x)dx = a\int f(x)dx$

4th :

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int adx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

Answer: $\int (x-4)\,(x+7)dx = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 28x + C$

19) $\int \frac{3x^2+2x}{x} dx$

20) $\int \frac{4x^2+5x}{x} dx$

1st : Create two fractions reduce / rewrite

2nd:

$\int (f(x) + g(x))dx = \int f(x)dx \quad + \int g(x)dx$

3rd: $\int af(x)dx = a \int f(x)dx$

4th :

Power Rule: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int adx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

Answer: $\int \frac{4x^2+5x}{x} dx = 2x^2 + 5x + C$

21) $\int 3e^x dx$

22) $\int 6e^x dx$

1st: $\int af(x)dx = a\int f(x)dx$

2nd: "e" Rule $\int e^x dx \ = e^x + C$

Answer: $\int 6e^x dx = 6e^x + C$

23) $\int -\frac{1}{2}e^{x}dx$

24) $\int -\frac{4}{5}e^{x}dx$

1st: $\int af(x)dx = a\int f(x)dx$

2nd: "e" Rule $\int e^{x}dx \quad = e^{x} + C$

$answer$: $-\frac{4}{5}e^{x} + C$

25) $\int \frac{7}{x} dx$

26) $\int \frac{3}{x} dx$

1st: Rewrite with -1 exponent

2nd: $\int af(x)dx = a\int f(x)dx$

3rd: "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

Answer: $\int \frac{3}{x} dx = 3ln|x| + C$

27) $\int \frac{-4}{x} dx$

28) $\int \frac{-2}{x} dx$

1st: Rewrite with -1 exponent

2nd: $\int af(x)dx = a\int f(x)dx$

3rd: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

$answer$: $\int \frac{-2}{x} dx = -2ln|x| + C$

29) $\int 3x^{-1}dx$

30) $\int 2x^{-1}dx$

1st : $\int af(x)dx = a\int f(x)dx$

2nd : "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

Answer: $\int 2x^{-1}dx = 2ln|x| + C$

31) $\int \frac{3}{5} x^{-1} dx$

32) $\int \frac{2}{3} x^{-1} dx$

1st : $\int a f(x) dx = a \int f(x) dx$

2nd : "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Answer: $\int \frac{2}{3} x^{-1} dx = \frac{2}{3} ln|x| + C$